Edge Detection using Mathematical Morphology

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Outline

- Introduction to Mathematical Morphology
- The Structuring Element
- Basic and Composite Operations
- Morphological Edge Detection
- Reduced Noise Morphological Edge Detection
- Other Edge Detectors
- Real-Time Edge Detection

Mathematical Morphology

- Developed by Matheron and Serra at L'Ecole des Mines in Paris.
- Uses set theory image analysis
- Can be used to find boundaries, skeletons, etc.
- Can be used for many pre and post image processing techniques
- Relies on two basic operations used to shrink and expand image features
- Originally used on binary images

Mathematical Morphology

- Two Basic Operations
  - Dilation (expands features)
  - Erosion (shrinks features)
- Composite Relations
  - Opening
  - Closing
- Operations
  - Edge Detection
  - Thinning
  - Thickening
  - Etc.

Structuring Element

- Small Binary Image
  - 3x3, 5x5, 7x7, etc.
- Swept over image
- Typical elements
  - Square
  - Disk
  - Ring
- Origin used as index pixel

1	1	1
1	1	1
1	1	1

Square (3x3)

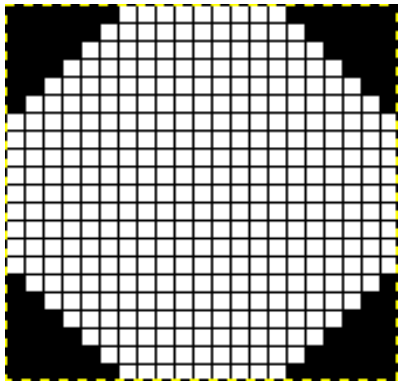
	1	1	1	
1				1
1				1
1				1
	1	1	1	

Ring (5x5)

	1	1	1	
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
	1	1	1	

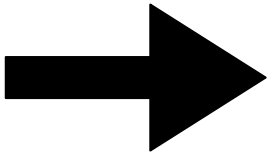
Disk (5x5)

Dilation



Structuring Element

Original Image



Dilated Image

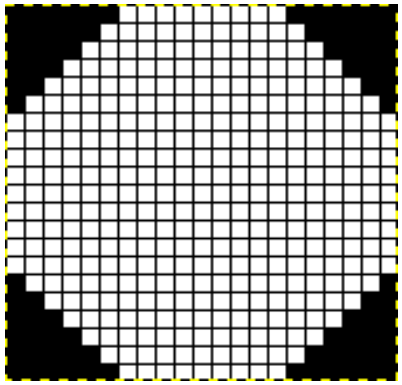
Dilation

- Expands features
- Structuring element swept over image
- Index pixel set to maximum found within structuring element

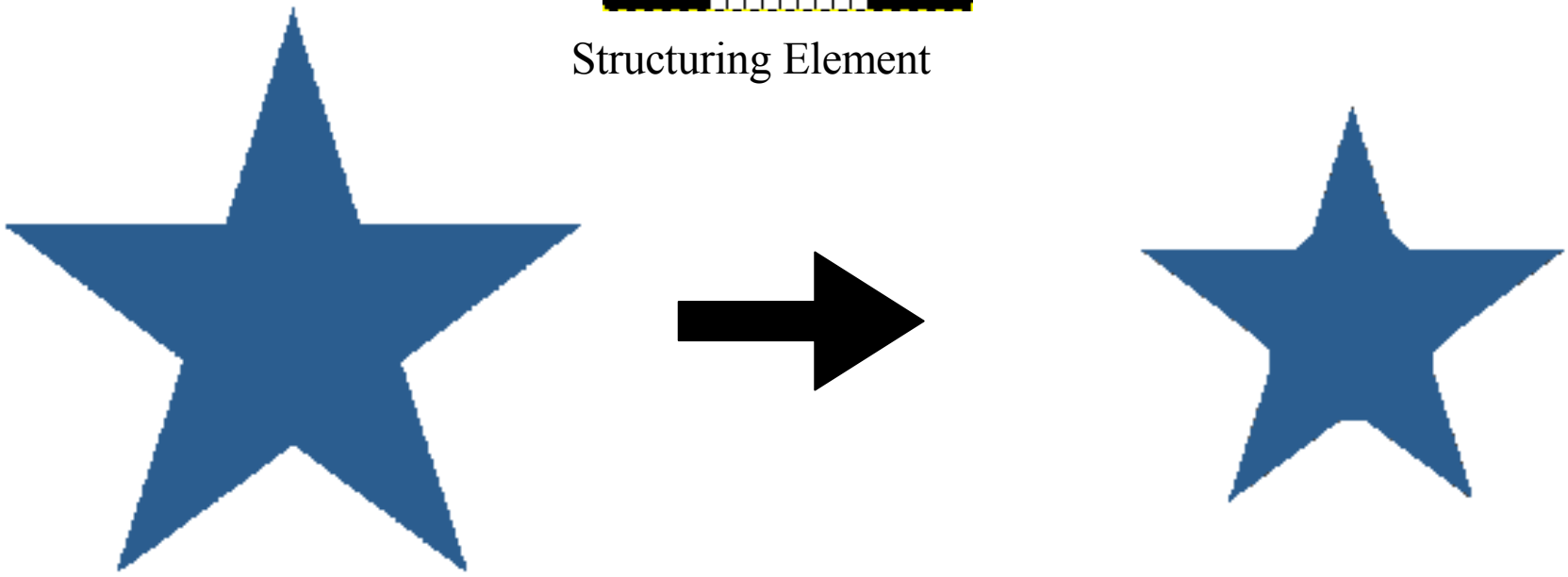
$$A \oplus B = \max_{[j,k] \forall B} \{a[m-j, n-k] + b[j,k]\}$$

Where,
A – Original Image
B – Structuring Element

Erosion



Structuring Element



Original Image

Eroded Image

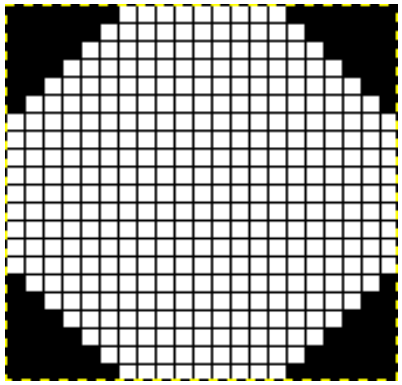
Erosion

- Reduces features
- Structuring element swept over image
- Index pixel set to minimum found within structuring element

$$A \ominus B = \min_{[j,k] \forall B} \{a[m-j, n-k] - b[j,k]\}$$

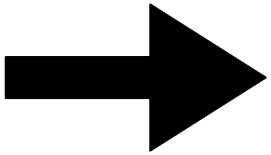
Where,
A – Original Image
B – Structuring Element

Opening



Structuring Element

Original Image



Opened Image

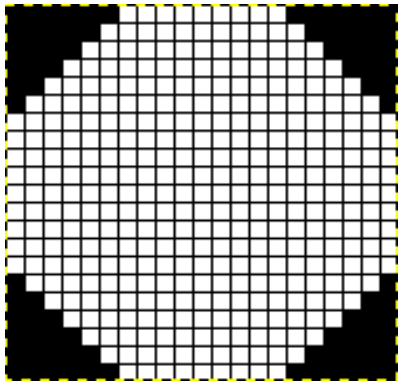
Opening

- Function of dilation and erosion
- Structuring element rolled along inner boundary

$$A \circ B = (A \ominus B) \oplus B$$

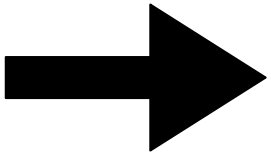
Where,
A – Original Image
B – Structuring Element

Closing



Structuring Element

Original Image



Closed Image

Closing

- Function of erosion and dilation
- Structuring element rolled along outer boundary

$$A \cdot B = (A \oplus B) \ominus B$$

Where,
A – Original Image
B – Structuring Element

Morphological Edge Detection

- Several edge detection techniques using morphology.
  - Erosion Residue Edge Detector
  - Dilation Residue Edge Detector
  - Morphological Gradient Edge Detector
  - Reduced noise Morphological Gradient Edge Detector

Morphological Gradient Edge Detection

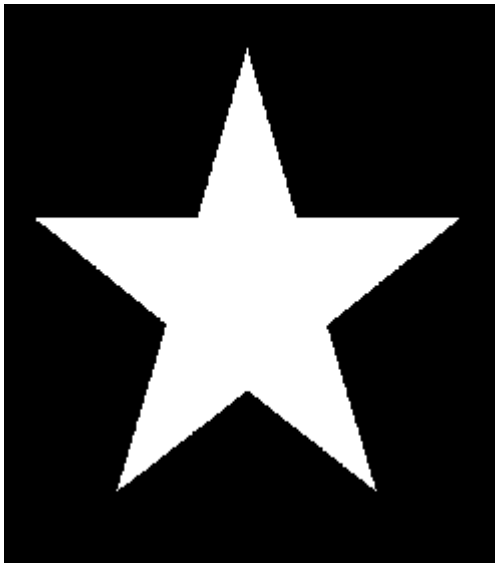
- Based on eroded and dilated image
- Provides good edge detection
- Should be followed by threshold for most applications

$$E_G(A) = (A \oplus B) - (A \ominus B)$$

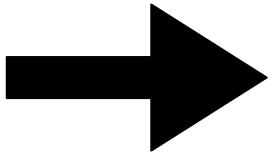
Morphological Gradient Edge Detection

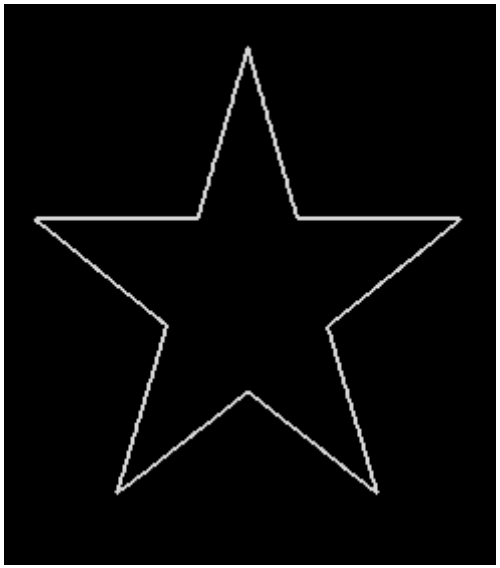
	1	
1	1	1
	1	

Structuring Element



Original Image



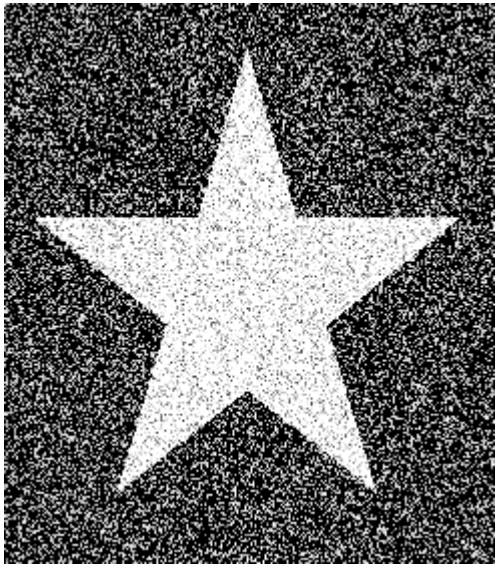


Edge Detection

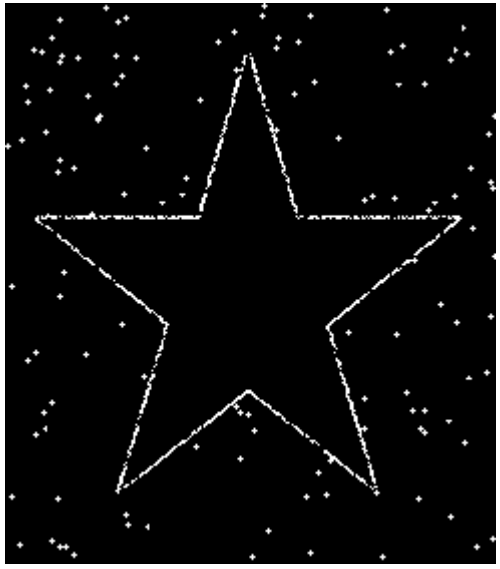
Morphological Gradient Edge Detection (noisy image)

	1	
1	1	1
	1	

Structuring Element



Original Noisy Image



Edge Detection (after thresholding)

Reduced Noise Morphological Edge Detection

- Proposed by Zhao et al. [1]
- Reduced noise
- Opening and closing to pre-process image filtering noise
- Close *M* to smooth

$$E_{reduced\text{-}noise}(A) = (M \cdot B) \oplus B - M \cdot B$$

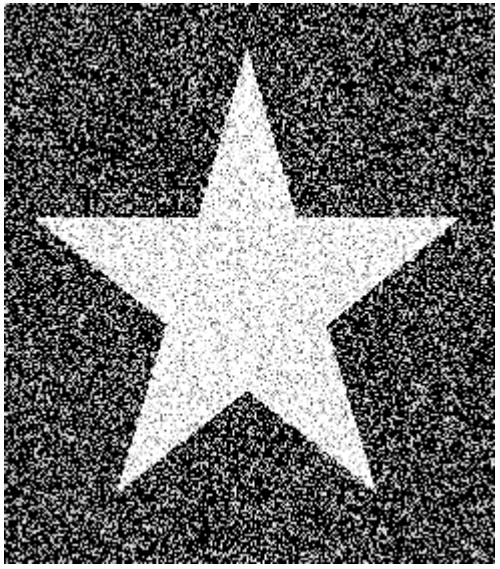
where,

$$M = (F \cdot B) \circ B$$

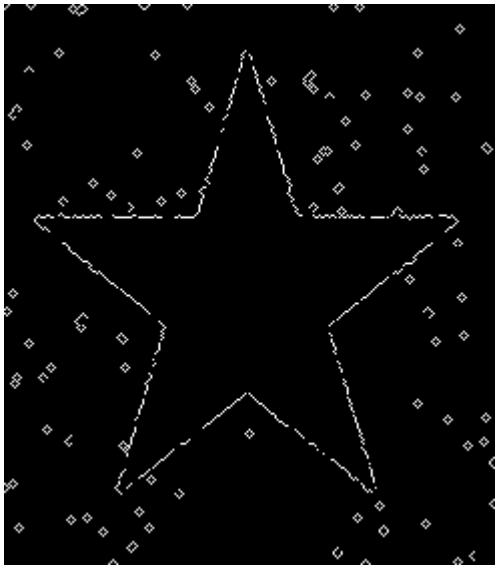
Reduced Noise Morphological Gradient Edge Detection

	1	
1	1	1
	1	

Structuring Element



Original Noisy Image



Edge Detection (after thresholding)

Other Edge Detection Algorithms

- Some very well known edge detection algorithms include
  - Prewitt
  - Sobel
  - Canny
- These techniques detect edges based on directional gradients

Prewitt and Sobel Edge Detection Algorithms

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad G_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

Sobel Convolution Kernel

$$G_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad G_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

Prewitt Convolution Kernel

The gradient magnitude is given by:

$$G = \sqrt{G_x^2 + G_y^2}$$

and can be approximated as follows:

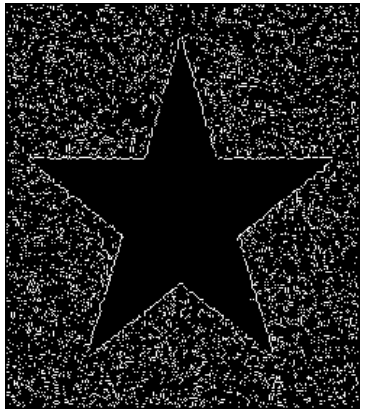
$$G \approx |G_x| + |G_y|$$

Canny Edge Detection Algorithm

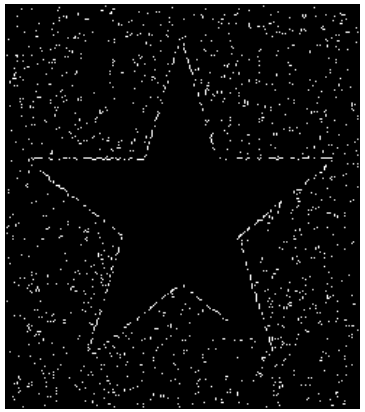
- More complex than Sobel, Prewitt and Morphological Gradient
- Six Step Process
  - Gaussian Mask (noise removal)
  - Sobel Operator to find edge strengths
  - Edge directions are computed
  - Edge directions are descretized
  - Non-maxima suppression
  - Hysteresis

Comparison of Algorithms on a Noisy Image

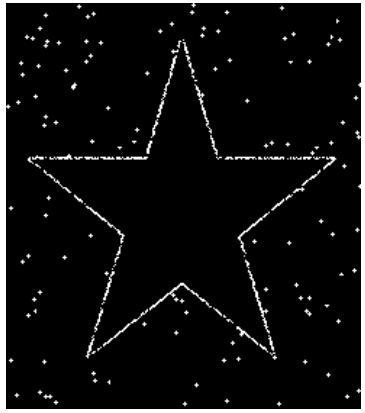
Original Image



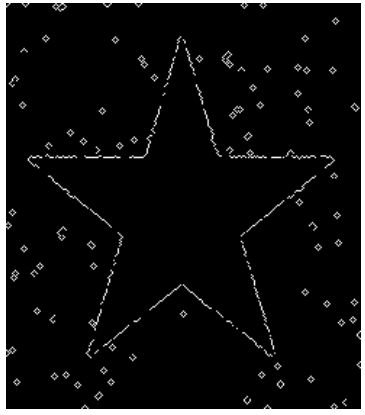
Prewitt



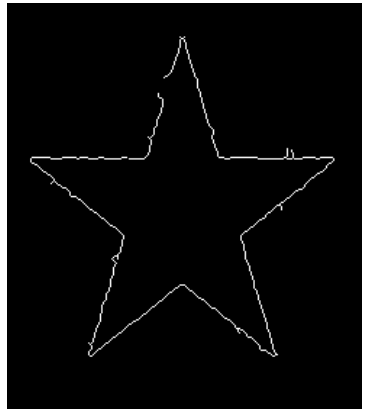
Sobel



Morphological Gradient



Reduced Noise Morphological Gradient



Canny

Real Time Edge Detection

- Edge detection can be performed in real-time.
- Can easily be done using the Morphological Gradient, Sobel or Prewitt algorithms
- A storage element is essential
- Delay will be imminent
- Must be performed on gray level image information

Questions?

Referencs

[1] S. Aksoy, “Binary Image Analysis”, Internet: http://cs.bilkent.edu.tr/~saksoy/courses/cs484/slides/cs484_binary.pdf, 2007, [cited 2007 Apr 1].

[2] William Green, “Edge Detection Tutorial.” Internet: http://www.pages.drexel.edu/~weg22/edge.html, 2002, [cited 2007 Apr 5].

[3] Zhao Yu-qian, Gui Wei-hua, Chen Zhen-cheng, Tang Jing-tian and Li Ling-yun, “Medical Images Edge Detection Based on Mathematical Morphology,” in Proc. IEEE-EMBS, 2005, pp. 6492-6495.

[4] Robyn Owens, “Mathematical Morphology.” Internet: http://homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/OWENS/LECT3/node3.html, 1997, [cited 2007 June 13].

[5] Bob Fisher, Simon Perkins, Ashley Walker, Erik Wolfart, “Sobel Edge Detector.” Internet: http://www.cee.hw.ac.uk/hipr/html/sobel.html, 1994, [cited 2007 June 13].