A 2DLNS based Multiplier and Accumulator (MAC) unit

Mahzad Azarmehr

Supervisor: Dr. R. Muscedere

University of Windsor

Electrical and Computer Engineering

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Introduction

- **Desired characteristics of a number system used in DSP:**
  - More error-free mapping approximations
  - Less complexity of arithmetic operations
  - Smaller size of corresponding representations
  - More accurate representation of smaller values

MDLNS (Representation)

- A representation of the real number X , in the form:

$$x = \sum_{i=1}^{n} s_i \prod_{j=1}^{b} p_j^{e_j^{(i)}}$$

- where s_i is sign, p_j can be real, and $e_j^{(i)}$ are integers, is called an n digit multi-dimensional logarithmic representation of X

- b is the number of bases used (at least two) and the first one, p_1 will always be assumed to be 2

MDLNS (Properties)

- A large reduction in hardware
- An attendant reduction in complexity of the hardware
- Ability to choose the best possible representation for each application

MDLNS (Calculations)

- **Multiplication and Division**

Given a single-digit representation of $x=\{s_x, a_x, b_x\}$ and $y=\{s_y, a_y, b_y\}$:

$$x.y = \{ s_x \text{ xor } s_y , a_x + a_y , b_x + b_y \}$$

$$x \div y = \{ s_x \text{ xor } s_y , a_x - a_y , b_x - b_y \}$$

MDLNS (Calculations)

- **Addition and Subtraction**

$$2^{a}x \,.\, D^{b}x + 2^{a}y \,.\, D^{b}y = (2^{a}x \,.D^{b}x).(1 + 2^{a}y^{-a}x \,.\, D^{b}y^{-b}x\ )$$

$$\approx (2^{a}x \,.D^{b}x).\Phi(a_y - a_x\ ,\ b_y - b_x)$$

$$2^{a}x \,.\, D^{b}x - 2^{a}y \,.\, D^{b}y = (2^{a}x \,.D^{b}x).(\ 1 - 2^{a}y^{-a}x \,.\, D^{b}y^{-b}x\ )$$

$$\approx (2^{a}x \,.D^{b}x).\Psi(a_y - a_x\ ,\ b_y - b_x)$$

The operators Φ and Ψ are lookup tables that store the precomputed 2DLNS values.

MDLNS (Conversions)

$$x = \sum_{i=1}^{n} s_i \cdot 2^{a_i} \cdot D^{b_i}$$

- If R bits are considered to represent second base (D) exponent, $b_i = \{-2^{R-1}, \ldots, 2^{R}-1\}$ and range of a_i is determined regarding to D and Q digit equivalent binary data
- b is used as an index address to a LUT to find a pseudo-floating point representation for D^b and finally a normalized binary expression for the 2DLNS representation

MDLNS (Conversions)

- The pseudo-floating point representation is in the form $\mu(b) \cdot 2^{\varepsilon(b)}$, where $\mu(b)$ is the mantissa (real) and $\varepsilon(b)$ is the exponent (integer).

$$x = s \cdot \mu(b) \cdot 2^{(a + \varepsilon(b))}$$

- To convert a binary representation to a single-digit 2DLNS, the normalized mantissa, $\mu(b)$, would be the input to the LUT.

Conversion Look-up Table

- Conversion LUT for D = 3, R = 3, C = 10

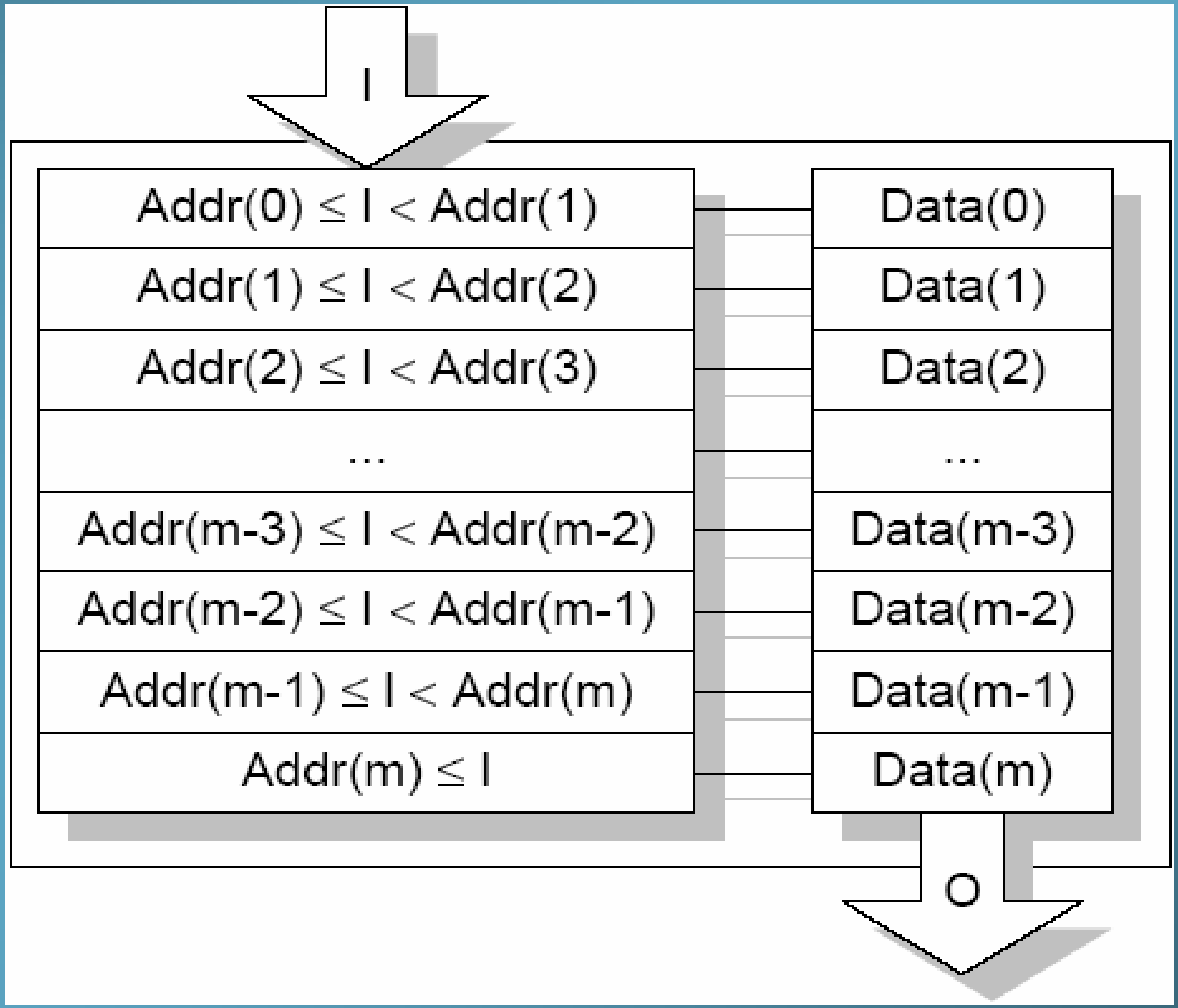
Input	Output	
$\mu(D^b)$ (base 2)	$\varepsilon(D^b)$	b
1.0000000000	0	0
↓	?	?
1.0010000000	3	2
↓	?	?
1.0010111101	-5	-3
↓	?	?
1.0101010101	-2	-1
↓	?	?
1.1000000000	1	1
↓	?	?
1.1001010010	-7	-4
↓	?	?
1.1011000000	4	3
↓	?	?
1.1100011100	-4	-2
↓	?	?

Range Addressable Look-Up Table (RALUT)

- Size of LUT can be reduced by a decrease in number of rows from 2^C (which C is the number of decimal point digits of mantissa) to $2^R + 1$

- A RALUT differs from the classic LUT by changing the address decoder system to match on a range of values rather than exact values

RALUT Structure



Range Addressable Look-Up Table (RALUT)

- Most of the designs which are used in MDLNS circuits can be efficiently implemented with RALUTs.
- The proper second base (optimal base) should be selected in accordance to the specific design consideration.
- Size of RALUTs and their contents should be adjusted based on optimal bases and necessary precisions.

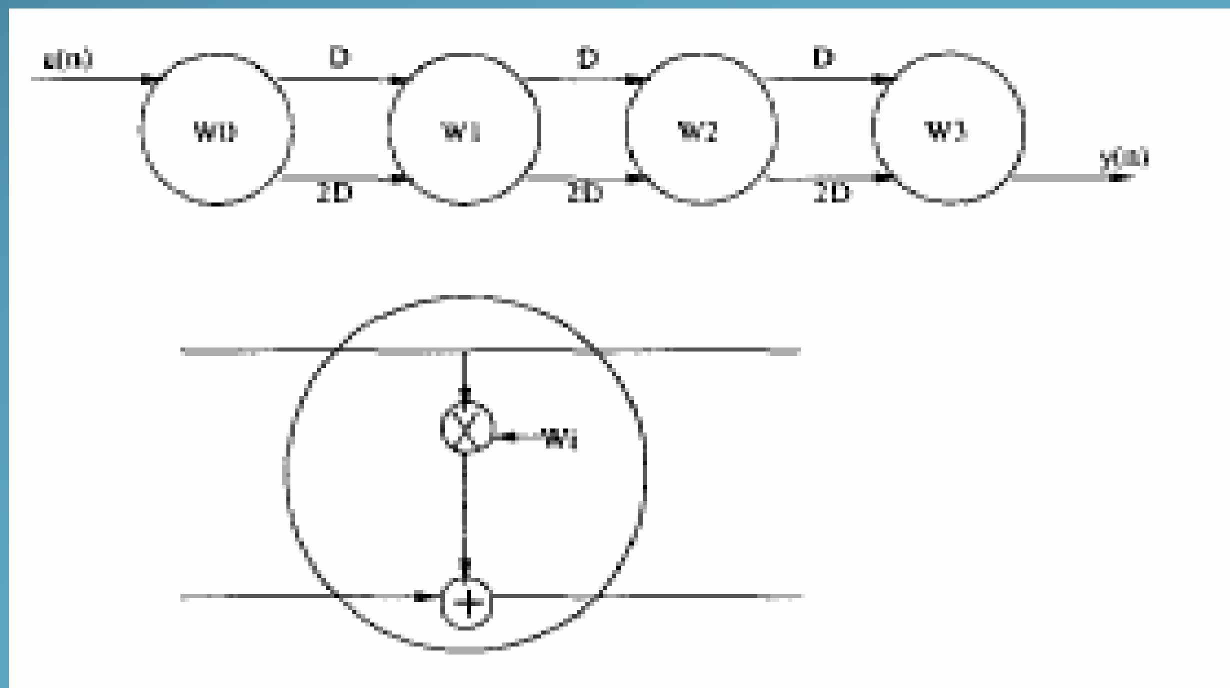
Multiplier and Accumulator (MAC) unit

- A MAC multiplies corresponding elements of two sequences of numbers $\{X_i\}$ and $\{Y_i\}$ and accumulates the sum of the products:

$$P = \sum_i X_i . Y_i$$

- The implementation of a MAC needs intensive computation and consumes much resources. There is always a traditional trade off of size versus speed.

MAC in FIR filter



$$y[n+1] = y[n] + x(n) \cdot w(n)$$

MAC unit (Specifications)

- Coefficients :

  2-digit 2DLNS numbers (D = 1.28308348549366 , B = 6 , R = 3)

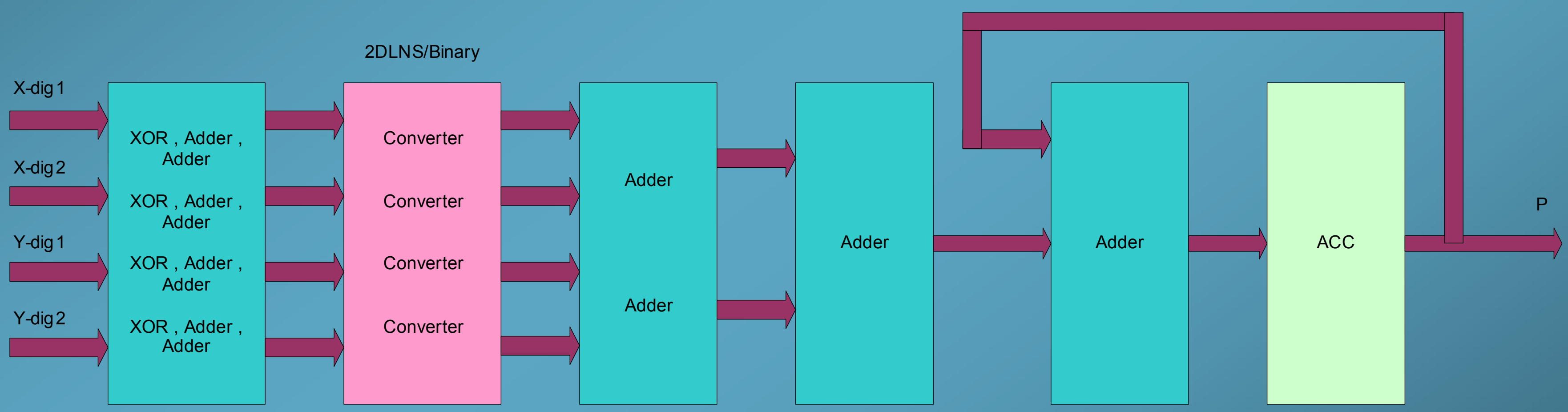
- Input data :

  2-digit 2DLNS numbers (D = 1.28308348549366 , B = 6 , R = 5)

- There are four partial products for each pair of numbers.

- Multiplications are performed in 2DLNS, but partial products are converted to Binary to be added.

MAC unit (Structure)



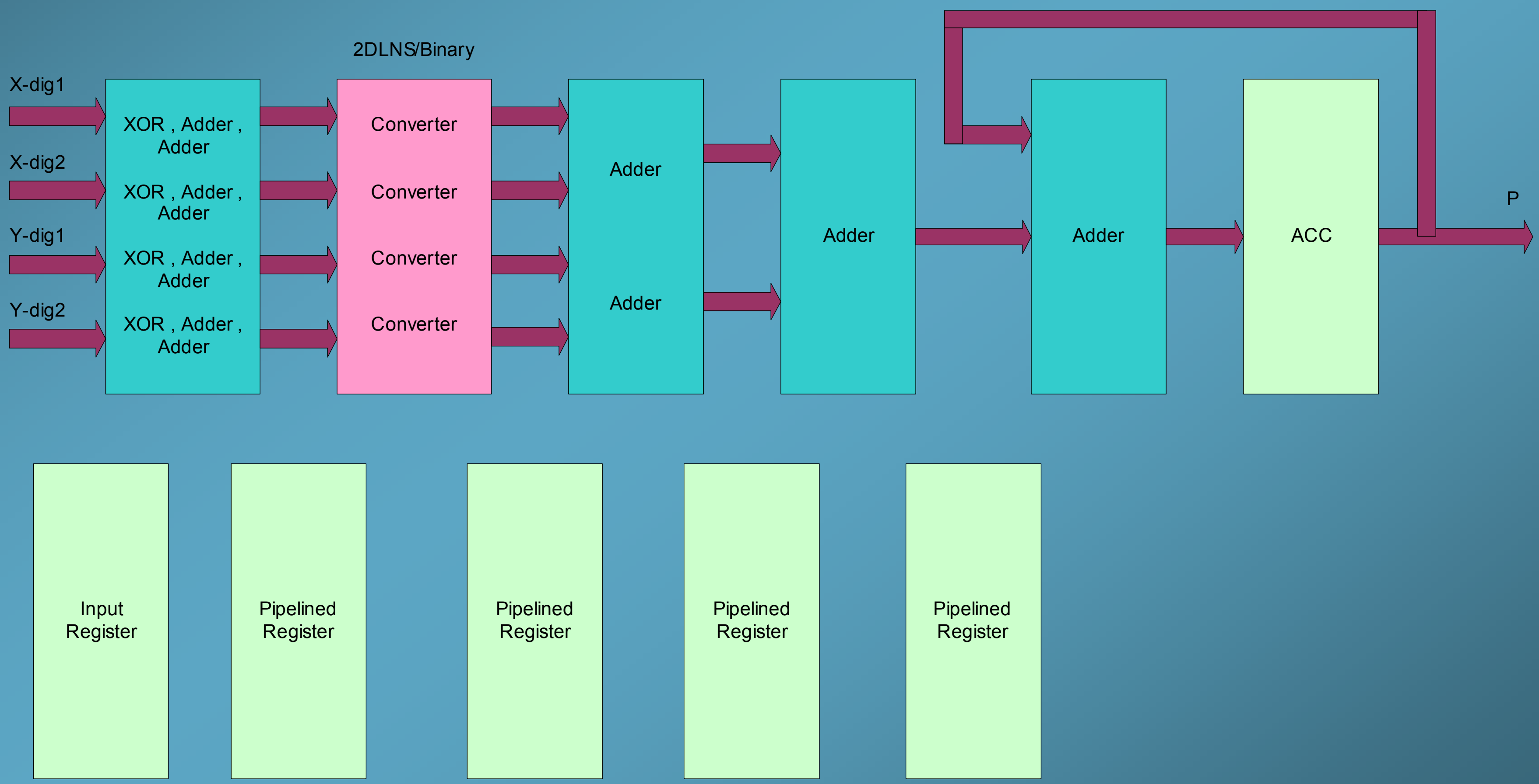
MAC unit (Pipelined)

- The time taken to complete processing one pair of inputs is the sum of the delays for all stages. This delay can be avoided by pipelining the MAC.

- The advantage of pipelined approach is that the clock period can be reduced to the slowest of the pipeline stages, rather than the total of their delays.

- The more efficiency for pipelining is the case that each stage performed in just one clock cycle.

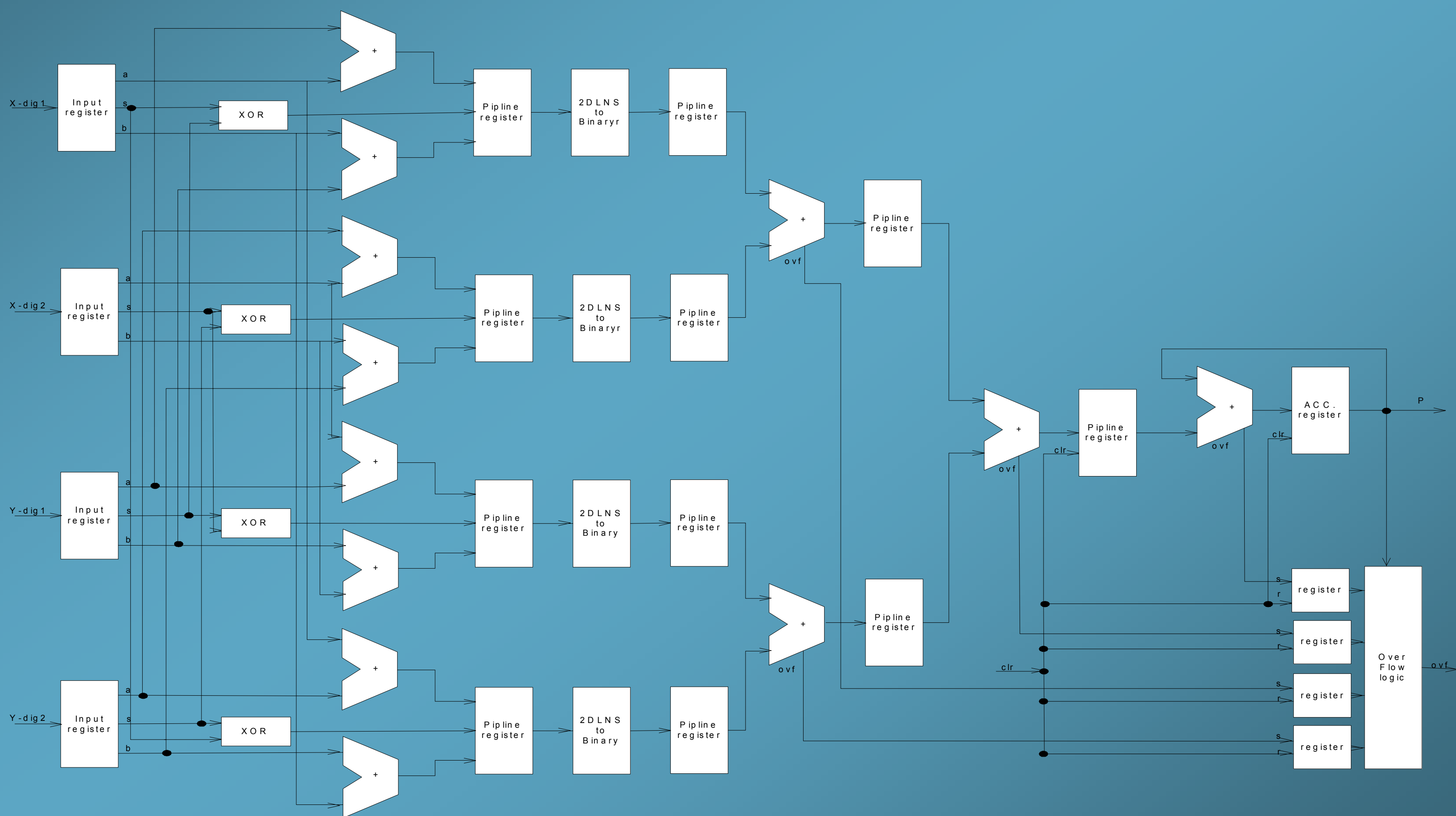
MAC unit (Pipelined Structure)



MAC unit (Overflow)

- Overflow in intermediate partial sums; an overflow flag must be set
- Overflow in the final accumulation (may be a transient condition);
  - Expansion the range used to represent result
  - An overflow flag must be set
- The final overflow signal is an OR combination of all overflow flags

MAC unit (RTL organization)



MAC unit (Reduced RALUT)

- Second base exponent (B) of Input data can be limited in range -12 to 12 instead of -16 to 15. Since intermediate sums can be shown by R = 5 instead of R = 6, a RALUT with almost half size can be used.

- This means to force data from order of 2^{-40} be limited to order of 2^{-37}.

- However, all possible representations should be checked beforehand in terms of acceptable mapping precision.

MAC unit (Synthesis Results)

Factor	**R = 6** **Clock-pulse = 10(ns)**	**R = 5** **Clock-pulse = 8(ns)**
Data required time (ns)	9.35	7.85
Data arrival time (ns)	-9.35	-7.85
Slack (ns)	0.00	0.00
Total cell Area (μm)^2	284496.5625	119642.1094
Total Dynamic Power (mw)	10.9670	9.8602

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Questions and Comments